Determination of the effects of temperature on the C_v value of CO_2

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ABSTRACT:

Results: The calculated observable C_v is the heat capacity at a constant volume. During the experiment the lab temperature was 19.5°C, the relative humidity was 47%, and the atmospheric pressure was 754.5mm Hg. The regression coefficients for speed of sound in air, N_2, room temp CO_2, and dry ice CO_2 were found for each experimental trial; samples of these plots are shown below.

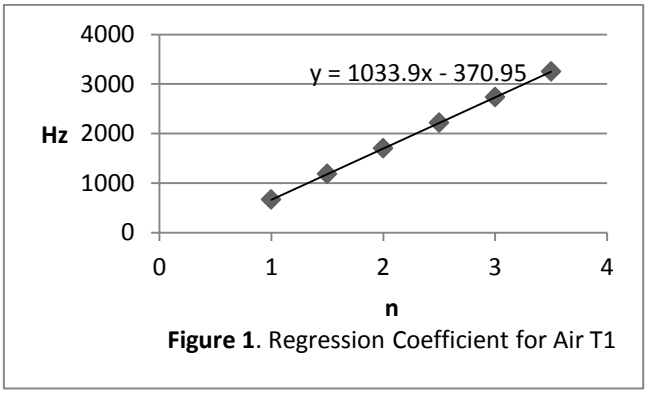

Figure 1. Regression Coefficient for Air T1

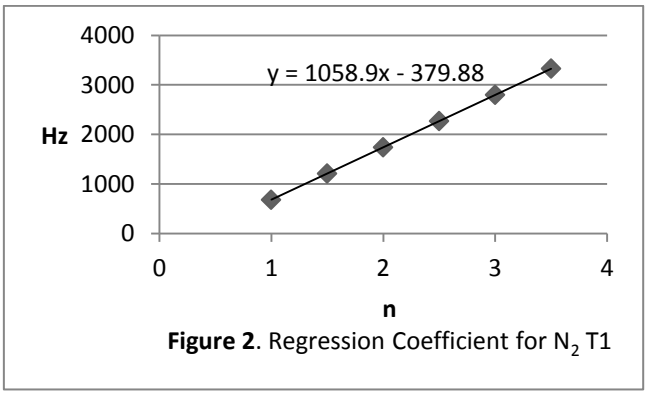

Figure 2. Regression Coefficient for N_2 T1

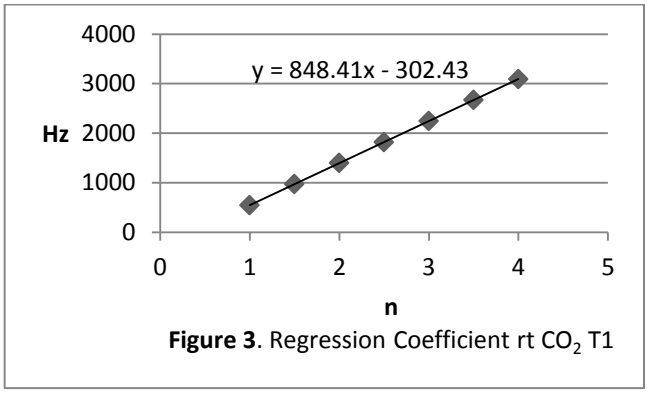

Figure 3. Regression Coefficient rt CO_2 T1

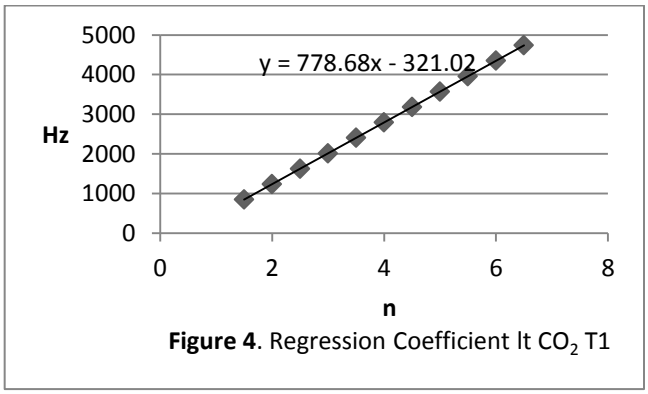

Figure 4. Regression Coefficient lt CO_2 T1

Table 1. Slopes and Errors for Regression Coefficients

	Trial 1	Trial 2	Trial 3
Air	1033.94+/-129.07	1032.83+/-128.35	1033.40+/-129.60
N_2	1058.88+/-124.95	1061.29+/-127.13	1060.77+/-126.31
Room Temp CO_2	848.41+/-91.25	850.16+/-91.34	849.33+/-92.56
Dry Ice CO_2	778.68+/-84.75	778.41+/-84.25	777.84+/-84.27

Table 2. Average Slope and Error for Air, N_2, room temp CO_2, and dry ice CO_2

	Average Slope
Air	1033.39 +/- 74.48
N_2	1060.31 +/- 72.82
Room temperature CO_2	849.30 +/- 52.95
Dry Ice CO_2	778.31 +/- 48.74

The error for the average slopes of the regression coefficient plots (shown in table 2) was calculated using the following equation $AveErr = (\frac{1}{3})\sqrt{errT_1^2 + errT_2^2 + errT_3^2}$ and the data from table 1. A literature value of the speed of sound in Nitrogen gas (un), 334 m/s [*1*], was corrected for temperature differences [*2*] with the equation

$$un = \sqrt{\frac{RTY}{M}} \qquad (1)$$

where R is the universal gas constant, T is the temperature in Kelvin (293.75 °K), Y is 7/5 [*2*], and M is the atomic mass (Kg) of Nitrogen. The calculation of the velocity of sound in nitrogen is important because un is one of the variables and sources of error in the calculation of C_v. After calculating the correct velocity (349.45 m/s), the length of the apparatus was calculated by

$$L = un/(\frac{df}{dn})_{N_2} \quad (2)$$

The calculated length of the tube was 0.330 m and the measured length of the tube was 0.248 m. The data from the nitrogen trials was used to calculate the length of the tube due to the fact that using both air and nitrogen in the calculation would cause error due to the use of different velocities for air and nitrogen in the equation. The calculation of tube length is important to the experiment as equation (2) will be used in the derivation of the C_v calculation.

The equation for C_v was derived from the following equation

$$u^2 = \frac{\gamma RT}{M} = \frac{\left(\frac{C_p}{C_v}\right)RT}{M} \quad (3)$$

Where M is the molar mass of CO_2 in Kg; the value for C_p can be converted into terms of C_v with the equation $C_p = C_v + R$ allowing equation (3) to be written in terms of C_v

$$C_v = \frac{R^2T}{\left(u_{CO_2}^2 * M\right) - RT} \quad (4)$$

The equation $u_{CO_2} = (\frac{df}{dn})_{CO_2} * L$ can be combined with equation (2) to produce the equation

$$u_{CO_2} = (\frac{df}{dn})_{CO_2} * \left(\frac{un}{\left(\frac{df}{dn}\right)_{N_2}}\right) \quad (5)$$

Equation (5) can then be combined with equation (4) to create the formula for C_v shown below

$$C_v = \frac{R^2 * T}{-RT + \frac{M\left(\frac{df}{dn}\right)_{CO_2}^2 * un^2}{\left(\frac{df}{dn}\right)_{N_2}^2}} \quad (6)$$

Equation (6) is then used to calculate the values of C_v for room temperature CO_2 and dry ice CO_2. The values of the variables used for each calculation, as well as the calculated values of C_v are shown in table 3.

Table 3. Values Used in the Calculation of C_v for room temp and dry ice CO_2

	Room Temp CO_2	Dry Ice CO_2
M	0.04401 Kg	0.04401 Kg
T	292.65 °K	202.15 °K
R	8.314 J/K*mol	8.314 J/K*mol
un	349.45 m/s	349.45 m/s
$(\frac{df}{dn})_{CO_2}$	849.30 s^{-1}	778.31 s^{-1}
$(\frac{df}{dn})_{N_2}$	1060.31 s^{-1}	1060.31 s^{-1}
C_v	19.93 J/K	11.50 J/K

Since the errors of $(\frac{df}{dn})_{N_2}$ and $(\frac{df}{dn})_{CO_2}$ for room temperature and dry ice CO_2 have already been calculated (see table 2), only the errors for T and un must be calculated before calculating the error for C_v. The error for temperature was calculated using the equation

$$Err(T) = 0.01 * T \quad (7)$$

as a base for the error calculation, resulting in a temperature error of +/- 2.927 °K for room temp CO_2 and +/- 2.022 °K for dry ice CO_2.

The error for the corrected speed of sound in nitrogen gas was calculated using equation (1) as the base of the error calculation. The partial derivative used in the calculation was taken with respect to T. The error of T used to calculate un error was found by applying equation (7) to the temperature for N_2 (293.75 °K). When the calculation was performed the

error for un was found to be +/- 1.747 m/s. The errors for the components of the C_v error are shown in table 4.

Table 4. Errors for the Components of C_v Error

	Room Temp CO_2	Dry Ice CO_2
$(\frac{df}{dn})_{CO_2}$	+/- 52.95	+/- 48.74
$(\frac{df}{dn})_{N_2}$	+/- 72.82	+/- 72.82
un	+/- 1.747	+/- 1.747
T	+/- 2.927	+/- 2.022

The data from table 4 was used in the equation

$$ErrC_v = Sqrt[(dC_v/d\left(\frac{df}{dn}\right)_{CO_2})^2 * (err(\frac{df}{dn})_{CO_2})^2 + (dC_v/d\left(\frac{df}{dn}\right)_{N_2})^2 * (err(\frac{df}{dn})_{N_2})^2 + (dC_v/d\,un)^2 * \left(err(un)\right)^2 + (dC_v/dT)^2 * \left(err(T)\right)^2] \quad (8)$$

to calculate the error of C_v for both room temperature CO_2 (+/- 12.60) and dry ice CO_2 (+/- 5.11).

The theoretical value of C_v was calculated with the equation

$$C_v = \left(\frac{5}{2}\right)R + C_{vib} \quad (9)$$

however, the value of C_{vib} had to be calculated before the theoretical value of C_v could be calculated, this was done using

$$C_{vib} = R * \sum_{i=1}^{i=4} \frac{\left(\frac{hcv_i}{KT}\right)^2 * e^{\frac{hcv_i}{KT}}}{(e^{\frac{hcv_i}{KT}}-1)^2} \quad (10)$$

where c is the speed of light in cm/s, T is temperature in °K, K is Boltzmann's constant, h is Planck's constant, and v_i is the vibrational mode(s) of CO_2. The values of the first four vibrational modes of CO_2 were taken form an outside source [3].

Table 5. v_i Values for CO_2 [3]

Vibrational Mode Number	Value (cm^{-1})
v_1	1330
v_2	667
v_3	667
v_4	667

The use of equation (10) resulted in a C_{vib} value of 11.415 for room temperature CO_2 and 4.963 for dry ice CO_2. These values were plugged into equation (9) to give the theoretical value for C_v at room temperature and in dry ice. The calculated C_v, C_v error, and theoretical C_v values for room temp and dry ice CO_2 are shown in table 6.

Table 6. Calculated C_v, C_v error, and theoretical C_v

	C_v	C_v error	Theo C_v
Room Temp CO_2	19.93	+/- 12.60	32.20
Dry Ice CO_2	11.50	+/- 5.11	25.75

The calculated value for C_v of room temp CO_2 was 19.93+/-12.60 J/K, though the value is much less than the theoretical value of 32.20 J/K the error value gave room temp C_v a maximum of 32.53 J/K, which includes the theoretical value. The calculated C_v value for dry ice CO_2 was 11.50+/- 5.11 J/K. Even with the error range included the maximum value for calculated dry ice C_v is less than the theoretical value of 25.75 J/K. This large discrepancy between the calculated and theoretical values for dry ice C_v is likely indicative of a procedural error that could have caused the collected data to be incorrect, resulting in a value of C_v that was too low.

References:

[1] CRC Handbook of Chemistry and Physics, 70th ed., Boca Raton, FL, 1990. E44&E47.

[2]ElectronicsTeacher.com.http://www.electronicsteacher.com/succeed-in-physical-science/sound/speed-of-sound-in-a-gas.php (accessed Oct 11, 2011).

[3]Kadhim,A.Uotechnology.edu.http://www.uotechnology.edu.iq/appsciences/physics/lecture/physics/four_class/spectral%20analysis/3.pdf (accessed Oct 10, 2011).