Determination of Heat Capacity via Sound Velocity Measurements

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Results

Various gases, air, nitrogen, and carbon dioxide, were used to find the tube length and ultimately the heat capacity of these gases. The tube length was varied and was done so as a measure of the gas flow in the system which will directly affect heat capacity and sound velocity. The measured tube length was found to be 44.5 cm. Ultimately the goal was to determine an independent measurement for the constant volume heat capacity for carbon dioxide, at two different temperatures through sound velocity experimentation.

Heat capacity was to be determined at constant volume using Equation (1) and Equation (2) can be used to determine u values based on the frequency of sound velocity waves.

$$C_v = \frac{R^2 T}{u^2 M - RT} (1)$$

Here R is the gas constant, T is the temperature in Kelvin, u is the velocity of sound and M is the molar mass.

$$\mathbf{u} = \left(\frac{df}{dn}\right) \mathbf{L} \tag{2}$$

Here df/dn is the slope, m. That equation was used to determine the length and sound velocity for carbon dioxide.

The velocity of sound for Nitrogen gas was found to be 334 m/s at 293 K [1]. However, the actual temperature for which we collected our Nitrogen data was $27 \circ C$. Using a ratio we found the corrected velocity for Nitrogen at our temperature to be 345 m/s.

Frequencies were plotted as a function of the n values versus being in phase and out of phase for each of the gases. A figure from each trial is included however the Nitrogen trial yielded the best results in that the average of the values was 44.5 the exact tube length.



Figure 1: Frequencies of sound measured in Nitrogen gas at 27°C-Trial 1.



Figure 2: Frequencies of sound measured in Carbon Dioxide at 20.1°C-Trial 3.



Figure 3: Frequencies of sound measured in Carbon Dioxide at approximately 54°C-Trial 3.

The slopes for all the gases can be seen in Table 1. Using this average slope data the heat capacities at constant

volume were found for carbon dioxide and those can be found in Table 2.

Table 1- Slopes for Frequency vs. Phase Plots

| Gas | Trial | Slope, m/s |
|------------------------|---------|----------------|
| Nitrogen | 1 | 646.87 |
| | 2 | 650.01 |
| | 3 | 643.20 |
| | Average | 647 <u>+</u> 3 |
| CO ₂ (298K) | 1 | 515.54 |
| | 2 | 515.70 |
| | 3 | 515.17 |
| | Average | 515 <u>+</u> 2 |
| CO ₂ (218K) | 1 | 470.81 |
| | 2 | 472.10 |
| | 3 | 470.92 |
| | Average | 471 <u>+</u> 2 |

From the average slope of the gases, the heat capacities at constant volume for the carbon dioxide trials were determine along with the associated error.

Table 2- Heat Capacities for CO₂

| Trial | Cv (J/mol K) |
|----------------------|---------------|
| CO _{2-298K} | 22 <u>+</u> 2 |
| CO _{2-218K} | 7 <u>+</u> 6 |

Error was found using the two equations below:

$$M_{avg} = \frac{(m1+m2+m3)}{3}$$
 (3)

By squaring the sum of these values, multiplying each by the standard deviation, and then finding the square root the error is found.

Other potential sources of error may have come from the oscilloscope not being calibrated

Theoretical values were found from data in the original packet provided by Dr. Pounds.

$$Cv = \frac{5}{2}R + R\sum$$

The theoretical value was found to be 28.5 J/mol K for our experimental temperature and 24.9J/mol K at 218 K. This implies there is error not affiliated with temperature with the slope.

The Monte Carlo method was used to calculate error [3]. Yielding regression statistics allowing for the exclusion of unfit data points and a near perfect (0.9989+) regression line. In the following figures the first two trials depicted in Figures 1-3 are depicted with error bars with standard error derived from the Monte Carlo method.



Figure 4: Frequencies of sound measured in Nitrogen gas at 27°C-Trial 1.



Figure 5: Frequencies of sound measured in Carbon Dioxide at 20.1°C-Trial 3.

This Monte Carlo method incorporates error from the oscilloscope that was not calibrated.

[1] *CRC Handbook of Chemistry and Physics*, 70th ed.; Weast, R.C., Lide, D.R., Astle, M.J., Beyer, W.H., Eds.; Boca Raton, FL, 1989; E-44

[2] Silbey, R.J.; Alberty, R.A.; Bawendi, M.G.
Rotational and Vibrational Spectroscopy. *Physical Chemistry*, 4th ed.; John Wiley & Sons: Hoboken, NJ, 2005;487

[3] Pounds, A.J. Private communication, 2011.