Rotating a Vector to the X-Axis

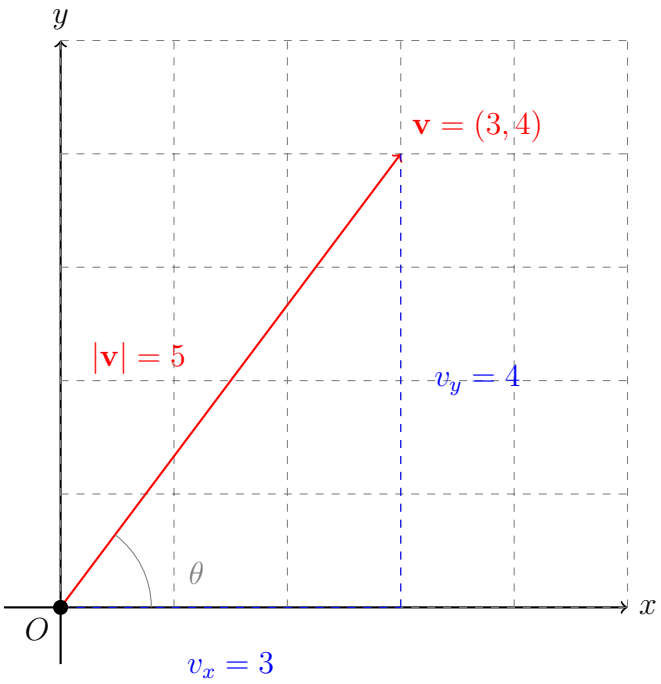
1 Problem Statement

Given a vector $\mathbf{v} = (v_x, v_y)$ in the xy-plane at the origin, find the rotation angle θ needed to align this vector with the positive x-axis.

Example: Rotate the vector $\mathbf{v} = (3, 4)$ to align with the x-axis.

2 Geometric Visualization

2.1 Before Rotation



3 Mathematical Solution

3.1 Step 1: Identify Components

From the vector $\mathbf{v} = (v_x, v_y) = (3, 4)$:

$$v_x = 3 \tag{1}$$
$$v_y = 4 \tag{2}$$

3.2 Step 2: Calculate Magnitude

Using the Pythagorean theorem:

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \tag{3}$$

3.3 Step 3: Determine Current Angle

The current angle θ from the x-axis is found using the arctangent:

$$\theta = \arctan 2(v_y, v_x) = \arctan 2(4, 3) \tag{4}$$

Calculation:

$$\sin(\theta) = \frac{v_y}{|\mathbf{v}|} = \frac{4}{5} = 0.8 \tag{5}$$
$$\cos(\theta) = \frac{v_x}{|\mathbf{v}|} = \frac{3}{5} = 0.6 \tag{6}$$
$$\theta = \arcsin(0.8) = 53.13^\circ = 0.9273 \text{ rad} \tag{7}$$

3.4 Step 4: Rotation Needed

To align the vector with the positive x-axis (angle $= 0^\circ$), we must rotate by:

$$\text{Rotation angle} = -\theta = -53.13^\circ \tag{8}$$

The negative sign indicates a **clockwise** rotation (negative angle direction).

4 General Formula

For any vector $\mathbf{v} = (v_x, v_y)$:

Rotation angle to align with x-axis:

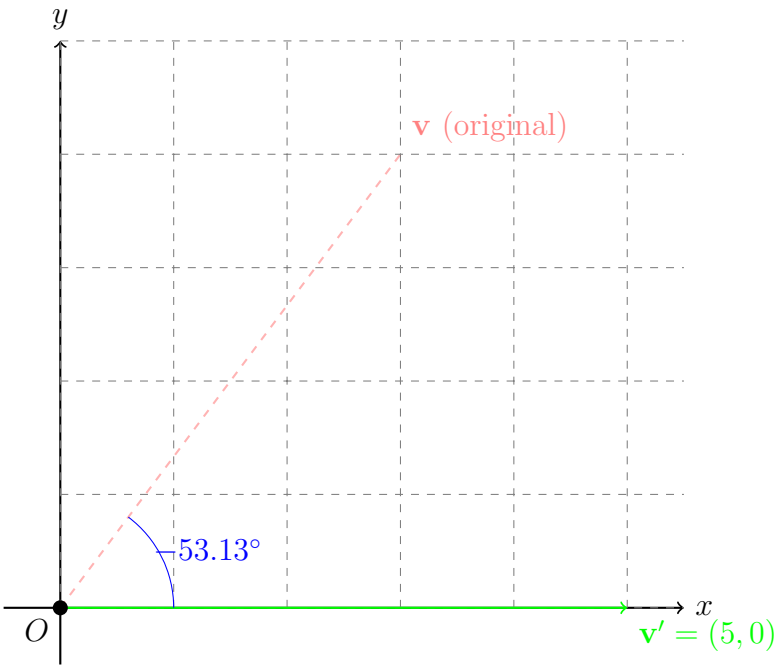
$$\theta_{\text{rotate}} = -\arctan 2(v_y, v_x) \tag{9}$$

Or equivalently:

$$\theta_{\text{rotate}} = \arctan 2(-v_y, v_x) \tag{10}$$

5 Visualization of the Rotation

5.1 Rotated Vector (After Rotation by $-53.13°$)



Result: After rotating by $-53.13°$, the vector aligns with the positive x-axis as $\mathbf{v}' = (5, 0)$.

6 The Rotation Matrix

The rotation matrix that rotates a vector by angle θ is:

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \tag{11}$$

For our rotation of $\theta_{\text{rotate}} = -53.13°$:

$$R(-53.13°) = \begin{pmatrix} \cos(-53.13°) & -\sin(-53.13°) \\ \sin(-53.13°) & \cos(-53.13°) \end{pmatrix} = \begin{pmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{pmatrix} \tag{12}$$

6.1 Verification

Apply the rotation matrix to the original vector:

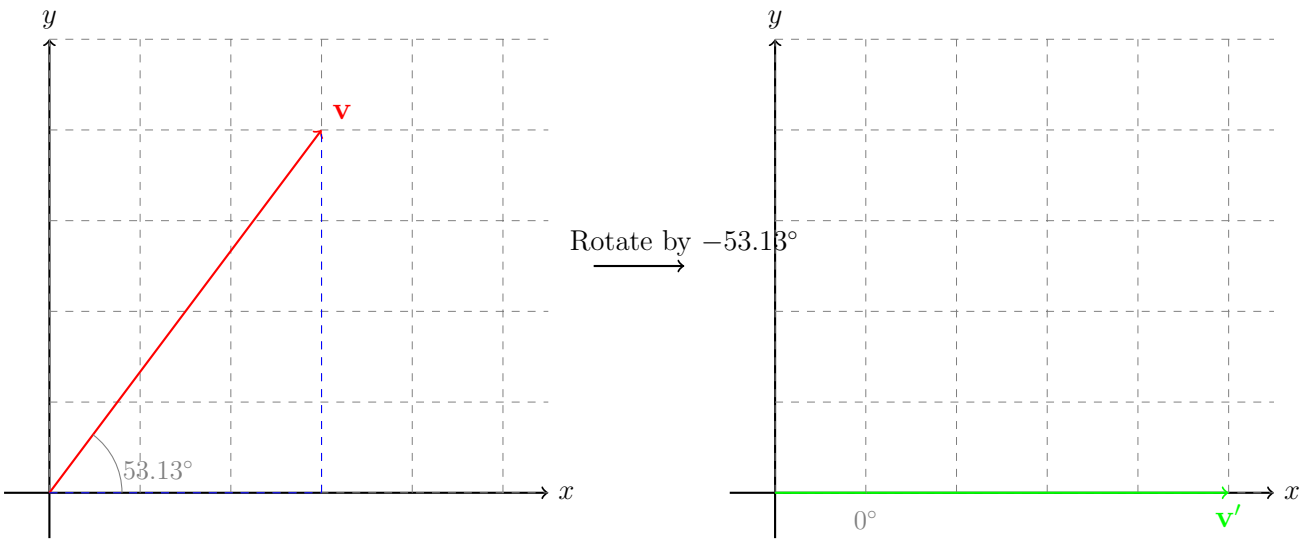
$$\mathbf{v}' = R(-53.13°) \cdot \mathbf{v} = \begin{pmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{13}$$

$$v'_x = 0.6 \times 3 + 0.8 \times 4 = 1.8 + 3.2 = 5 \tag{14}$$

$$v'_y = -0.8 \times 3 + 0.6 \times 4 = -2.4 + 2.4 = 0 \tag{15}$$

Result: $\mathbf{v}' = (5, 0)$ ✓ Aligned with the x-axis!

7 Side-by-Side Comparison



Before: $\mathbf{v} = (3, 4)$ After: $\mathbf{v}' = (5, 0)$

8 Summary

Property	Original	Rotated
Vector	$(3, 4)$	$(5, 0)$
Magnitude	5	5
Angle from x-axis	53.13°	0°
Rotation applied	—	−53.13°

9 Key Takeaways

- The rotation angle needed is the **negative** of the vector's current angle from the x-axis
- Formula: $\theta_{\text{rotate}} = -\arctan 2(v_y, v_x)$
- The magnitude of the vector is **preserved** by rotation
- Apply the rotation matrix $R(\theta)$ to transform the vector
- The result is the same vector, now pointing along the positive x-axis