

To build the single transformation matrix we will take the matrix product of individual transformation matrices. Based on the work back from earlier chapters, we know that a pixel point can be mapped into the viewport using an equation like.

$$Px = Wx_{min} + (x - x_{min})(Vx_{max} - Vx_{min})/(x_{max} - x_{min})$$

Where  $Wx_{min}$  refers to the minimum x pixel point in the Graphics Window Minimum,  $Vx_{min}$  and  $Vx_{max}$  refer to the minimum and maximum x pixel positions in the Viewport window,  $x_{min}$  and  $x_{max}$  are the domain of the function and  $x$  is the independent variable  $x$  in the domain.

In a similar manner,

$$Py = Wy_{min} + (y - y_{min})(Vy_{max} - Vy_{min})/(y_{max} - y_{min})$$

where  $y_{min}$  and  $y_{max}$  refer to the range of the function.

Using this definition, we can construct the transformation matrices.

In[2]:= **T1 = {{1, 0, -xmin}, {0, 1, -ymin}, {0, 0, 1}}**

Out[2]= {{1, 0, -xmin}, {0, 1, -ymin}, {0, 0, 1}}

In[3]:= **{{1, 0, -xmin}, {0, 1, -ymin}, {0, 0, 1}}**

Out[3]= {{1, 0, -xmin}, {0, 1, -ymin}, {0, 0, 1}}

In[4]:= **S1 = {{(Vxmax - Vxmin) / (xmax - xmin), 0, 0},  
          {0, (Vymax - Vymin) / (ymax - ymin), 0}, {0, 0, 1}}**

Out[4]=  $\left\{ \left\{ \frac{Vx_{max} - Vx_{min}}{x_{max} - x_{min}}, 0, 0 \right\}, \left\{ 0, \frac{Vy_{max} - Vy_{min}}{y_{max} - y_{min}}, 0 \right\}, \{0, 0, 1\} \right\}$

In[5]:= **T2 = {{1, 0, Wxmin}, {0, 1, Wymin}, {0, 0, 1}}**

Out[5]= {{1, 0, Wxmin}, {0, 1, Wymin}, {0, 0, 1}}

In[6]:= **MatrixForm[T1]**

Out[6]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & -x_{min} \\ 0 & 1 & -y_{min} \\ 0 & 0 & 1 \end{pmatrix}$$

In[7]:= **MatrixForm[S1]**

Out[7]/MatrixForm=

$$\begin{pmatrix} \frac{Vx_{max} - Vx_{min}}{x_{max} - x_{min}} & 0 & 0 \\ 0 & \frac{Vy_{max} - Vy_{min}}{y_{max} - y_{min}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[8]:= **MatrixForm**[T2]

Out[8]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & Wxmin \\ 0 & 1 & Wymin \\ 0 & 0 & 1 \end{pmatrix}$$

And now to build to matrix product of these...

In[9]:= **P = T2.(S1.T1)**

$$\text{Out[9]= } \left\{ \left\{ \frac{Vxmax - Vxmin}{xmax - xmin}, 0, Wxmin - \frac{(Vxmax - Vxmin) xmin}{xmax - xmin} \right\}, \right. \\ \left. \left\{ 0, \frac{Vymax - Vymin}{ymax - ymin}, Wymin - \frac{(Vymax - Vymin) ymin}{ymax - ymin} \right\}, \{0, 0, 1\} \right\}$$

In[10]:= **MatrixForm**[P]

Out[10]/MatrixForm=

$$\begin{pmatrix} \frac{Vxmax - Vxmin}{xmax - xmin} & 0 & Wxmin - \frac{(Vxmax - Vxmin) xmin}{xmax - xmin} \\ 0 & \frac{Vymax - Vymin}{ymax - ymin} & Wymin - \frac{(Vymax - Vymin) ymin}{ymax - ymin} \\ 0 & 0 & 1 \end{pmatrix}$$

In[11]:= **Vec = {{x}, {y}, {1}}**

Out[11]= {{x}, {y}, {1}}

In[12]:= **MatrixForm**[P.Vec]

Out[12]/MatrixForm=

$$\begin{pmatrix} Wxmin + \frac{(Vxmax - Vxmin) x}{xmax - xmin} - \frac{(Vxmax - Vxmin) xmin}{xmax - xmin} \\ Wymin + \frac{(Vymax - Vymin) y}{ymax - ymin} - \frac{(Vymax - Vymin) ymin}{ymax - ymin} \\ 1 \end{pmatrix}$$

Comparing the last matrix with our initial equations, we see that the individual equations for the x and y pixel mapping agree with the initial equations.

In[13]:= **Wxmin = 0;**  
**Wxmax = 700;**  
**Wymin = 0;**  
**Wymax = 700;**  
**Vxmin = 100;**  
**Vxmax = 600;**  
**Vymin = 100;**  
**Vymax = 600;**  
**xmin = -70;**  
**xmax = 50;**  
**ymin = 20;**  
**ymax = 125;**  
**x = -25;**  
**y = 100;**

```
In[27]:= N[P.Vec]
```

```
Out[27]= {{187.5}, {380.952}, {1.}}
```