CSC/MAT 335 - EXAM 1 PRE-TEST
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Name KEY Section ___

Show all work and draw a box around numerical answers.

1. For the following calculation:

$$\frac{(12.65)(2.588)}{(13.01)} - 2.516$$

(a) compute the result using four digit rouding.

0.001

(b) compute the result using four digit chopping.

- 0.001

(c) is there any loss of precision in the final result? If so, how many digits are lost? If not, why not?

3 DIGITS OF PRECISION LOST

2. The Maclaurin series expansion of $\ln(x+1)$ truncated to second order with its associated error term is given in Eqn. 1.

$$M_2(x) = \underbrace{x - \frac{x^2}{2}}_{\text{expansion}} + \underbrace{\frac{x^3}{3(\xi(x)+1)^3}}_{\text{error term}} \tag{1}$$

(a) What is the value of $M_2(.25)$?

0.21875

(b) What is the error bound in $M_2(.25)$?

MIN: $X = 0$

$$\frac{(.25)^3}{3(0+1)^3} = 0.005$$

MAX: $X = 0.25$

$$\frac{(.25)^3}{3(.25+1)^3} = 0.003$$

(c) What is the absolute error in $M_2(.25)$ compared to $\ln(0.25+1)$?

$$|0.21875 - .223144| = 0.004$$

3. You want to determine the **negative** root for the function

$$f(x) = x^3 + 2x + 10$$

using the bisection method.

(a) Identify two values that you know bound the root.

$[-2, 0]$

(b) Using the bounds from part (a), carry out four iterations of the bisection algorithm to determine a reduced domain in which the root exists.

| $a$ | $f(a)$ | $b$ | $f(b)$ | $f\left(\frac{b-a}{2}\right)$ |
|---|---|---|---|---|
| -2 | -2 | 0 | 10 | 7 |
| | | -1 | 7 | 3.625 |
| | | -1.5 | 3.625 | 1.14063 |
| | | -1.75 | 1.1406 | -0.341797 |
| -1.875 | -.341797 | | | |

$[-1.875, -1.8125]$

(c) Restarting from one of your bounds in part (a), carry out four iterations of Newton's method to find an approximation to the root.

$$f(x) = x^3 + 2x + 10$$

$$f'(x) = 3x^2 + 2$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_0 = 0$

$x_1 = -5$

$x_2 = -3.376623$

$x_3 = -2.40294$

$x_4 = -1.95368$

$x_5 = -1.85225$

(d) For your equation

$$f(x) = x^3 + 2x + 10$$

i. recast the equation in the form $g(x)$ so it can be solved by the fixed point (or Picard's) method.

$$f(x) = x^3 + 2x + 10 + X$$

ii. write a function code segment (or a subroutine) that solves for the root using the fixed point (or Picard's) method to 10 times *IEEE* single precision (32 bit) machine tolerance.

4. *NVIDIA* introduced a half precision data type (16 bits total with 1 sign bit, 5 bits in the exponent, and 10 bits in the mantissa). This later became the IEEE 754 half-precision binary floating point format known as **binary16**. What is the machine number for $\pi$ in this format? Please note that the most significant and least significant bits are noted with an "m" and "l" respectively.

| | | |
|---|---|---|
| 1 | 10001 | 1000000001 |
| | m l | m l |

$$(-1)^s \, 2^{c-\beta} (1+f) \qquad \beta = 2^{5-1} - 1 = 15$$

$$\pi = 3.14159265359 \qquad \pi/2 = 1.5707963268$$

$$c - \beta = 1$$

$$c = 16$$

| | | BIT |
|---|---|---|
| | 0.5707963268 | |
| $(\frac{1}{2})^1$ | 0.5 | 1 |
| | 0.0707963268 | |
| $(\frac{1}{2})^2$ | 0.25 | 0 |
| $(\frac{1}{2})^3$ | 0.125 | 0 |
| $(\frac{1}{2})^4$ | 0.0625 | 1 |
| | 0.0082963268 | |
| $(\frac{1}{2})^5$ | 0.03125 | 0 |
| $(\frac{1}{2})^6$ | 0.015625 | 0 |
| $(\frac{1}{2})^7$ | 0.0078125 | 1 |
| | 0.0004838268 | |
| $(\frac{1}{2})^8$ | 0.00390625 | 0 |
| $(\frac{1}{2})^9$ | 0.001953125 | 0 |
| $(\frac{1}{2})^{10}$ | 0.0009765625 | 0 |

| s | c | f |
|---|---|---|
| 0 | 10000 | 1001001000 |